Experimental validation for the determination of particle positions by the correlation coefficient method in digital particle holography

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Received 29 April 2008; revised 11 August 2008; accepted 2 October 2008; posted 13 October 2008 (Doc. ID 95565); published 3 November 2008

The feasibility and the accuracy of the correlation coefficient (CC) method for the determination of particle positions along the optical axis in digital particle holography were verified by validation experiments. A translation system capable of high precision was used to move the particle objects by exact known distances between several different positions. The particle positions along the optical axis were calculated by the CC method and compared with their exact values to obtain the errors of the focus plane determination. The tested particles were two-dimensional (2D) dots in a calibration target along with different-sized glass beads and droplets that reflected and caused a three-dimensional (3D) effect. The results show that the CC method can work well for both the 2D dots and the 3D particles. The effect of other particles on the focus plane determination was also investigated. The CC method can locate the focus plane of particles with high precision, regardless of the existence of other particles. © 2008 Optical Society of America

OCIS codes: 090.0090, 090.1195.

1. Introduction

Digital holography can instantaneously capture the volumetric information of a measurement field. It does not require a chemical process, as in optical holography, and its benefits are high efficiency, simplicity, and real time analysis [1]. Based on the optical setup, holography is classified as either in-line or off-axis [2]. In-line digital holography is widely used because it has a simpler optical configuration.

Digital holographic particle image velocimetry (DHPIV) is a potential three-dimensional (3D) particle velocity measurement tool that was developed in the last decade. It is nearly real time and is capable of full-field measurements. The most important problem in DHPIV is the precision of the particle locations, namely, how to determine the accurate focus plane of the particle objects. Various methods have been suggested to determine the focus plane. Yu and Cai investigated a criterion based on a gradient computation [3], Dubois et al. introduced a method that uses the score of the integrated amplitude modulus [4], Lefebvre et al. showed that the maximum modulus of a wavelet transform was existed at the best focus plane [5], Zhang et al. located the focus plane by using the Gabor transform [6], Klysubun et al. recognized the 3D target position using two-dimensional (2D) scanning holographic correlation and Wigner analysis [7], and Kim et al. located the focus plane using Wigner distribution [8,9].

In optical holography, the correlation coefficient (CC) method was first presented by Choo and Kang [10]. This method was also verified in digital
holography with the use of simulation holograms and 2D dot objects that were investigated for real particle holography [11]. That study calculated the mean value of the focus planes of all the dots by the CC method, which was regarded as the exact plane of the 2D dots, and then the errors of the focus plane of the dots were obtained from this mean value. In a real 3D particle field (for example, spray droplets), particles cannot be considered as simple plane dots and the interaction of many particles may hinder the determination of the particle positions. Therefore, it is necessary to use some types of validation experiments that can check the accuracy of the focus plane determination by the CC method for particles in a real 3D field.

We used the translation system capable of high precision to move the particle objects by exact known distances between several different positions. The tested particles were the same 2D dots in a calibration target that were used in the previous research and different-sized glass beads and droplets that reflected and caused a 3D effect. The particle positions along the optical axis were calculated by the CC method, compared with the exact values, and then the errors of the focus plane determination were obtained. The effect of other particles on the focus plane determination was also investigated.

2. Focusing Method

A. Principle of Digital Holography

The procedure of digital holography, namely, hologram recording and object reconstruction, is similar to that of optical holography, but it has several obvious advantages to optical holography. The recording medium is a CCD camera instead of photographic films. The reconstruction of the recorded holograms is achieved by a computer and is called numerical reconstruction.

Digital holography is classified as either in-line or off-axis depending on the optical setup. In-line digital holography is widely used because it has a simple optical configuration. The concept of in-line digital holography is shown in Fig. 1. An expanded laser beam goes through the object field and the part of the beam diffracted by the objects and arriving at the recording surface is considered the object beam, while the beam arriving without any distortion is considered the reference beam. The superposition of the two beams creates an interference pattern on the CCD sensor. This diffraction can be described by the Fresnel–Kirchhoff integral [12]:

\[
R(\xi', \eta') = \frac{i}{\lambda} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, y) E_R(x, y) \frac{\exp(-i \frac{2\pi}{\lambda} \rho)}{\rho} \, dx \, dy.
\]

with

\[
\rho = \sqrt{(\xi' - x)^2 + (\eta' - y)^2 + d^2}.
\]

The coordinates in Eqs. (1) and (2) are shown in Fig. 2. \(R(\xi', \eta')\) and \(E_R(x, y)\) are the wave fields of the reconstruction image and the reference beam, respectively, \(h(x, y)\) is the hologram function, \(d\) is the distance between two adjacent planes, \(\lambda\) is the wavelength, and \(\rho\) is the distance between two corresponding points in these two adjacent planes.

The reconstruction image \([R(\xi', \eta')]\) can be obtained by either the Fresnel method or the convolution method [12]. In our previous analysis [13], the resolution of the reconstruction image by the convolution method is better than that by the Fresnel method. Therefore, the convolution method is used in this study and the mathematical expression of the convolution method is

\[
R(\xi', \eta') = F^{-1} \left\{ \frac{F[h(x, y \cdot E_R)]}{\lambda} \cdot \frac{\exp \left[ -i \frac{2\pi}{\lambda} \sqrt{(d^2 + (x - \xi')^2 + (y - \eta')^2) \right]}}{\sqrt{(d^2 + (x - \xi')^2 + (y - \eta')^2}}} \right\},
\]

where \(F[]\) and \(F^{-1}[]\) are the Fourier transform and the inverse Fourier transform, respectively. In the present study, the reference beam, \(E_R(x, y)\), is 1 because it is the normal plane wave.

B. Correlation Coefficient Method

The correlation coefficient of the images is a number between 0 and 1, which measures the degree to which two images are similar. We have a correlation coefficient of 1 if there is perfect similarity. A value of 0 means that there is no similarity between the two images. The correlation coefficient is defined as

\[
\text{Correlation Coefficient} = \frac{\sum (I_1 - \bar{I}_1)(I_2 - \bar{I}_2)}{\sqrt{\sum (I_1 - \bar{I}_1)^2 \sum (I_2 - \bar{I}_2)^2}}
\]

where \(I_1\) and \(I_2\) are the two images being compared, \(\bar{I}_1\) and \(\bar{I}_2\) are the means of the two images, and \(\sum\) denotes the summation over all pixels.

Fig. 1. (Color online) Optical set-up of in-line digital holography.

Fig. 2. Coordinate system.
where \( m \) and \( n \) are the pixel indices, \( F \) and \( G \) are the images, and \( \bar{F} \) and \( \bar{G} \) are the average gray values of each image.

Figure 3 shows how to locate the focus plane using the correlation coefficient method. \( \Delta z \) is a reconstruction interval and \( \Delta C_z \) is a correlation interval. Along a certain distance, a series of particle images are reconstructed slice by slice with the reconstruction interval (\( \Delta z \)). Then, the correlation coefficient at an arbitrary plane along the optical axis is calculated, as is shown in Fig. 3. Two symmetrical images taken from each side from the focus plane should be the most similar, so the value of the correlation coefficient at the focus plane should be a maximum.

3. Validation Experiments

Validation experiments shown in Fig. 4 were conducted to check the accuracy of the focus plane determination by the CC method for particles in a real 3D field. At first, a CCD camera was put at position 1 (P1) and a hologram of an object was obtained. Then, the CCD was moved to P2 and P3, consecutively taking holograms of the object at each position. In this study, the translation system made by Physik Instrumente (PI) with translation stages (M-531) and control software (PI motion controller) was used to move the CCD accurately. The minimum resolution of the CCD movement by this translation system is 33 nm. \( \Delta d_1 \) and \( \Delta d_2 \) were then determined by the control software, which has enough precision for this study. The focus planes of these positions (\( d_{c1}, d_{c2}, \) and \( d_{c3} \)) are shown in Fig. 4 and were calculated using the CC method. The errors of these focus planes cannot be checked because the exact object distances are unknown, but \( \Delta d_1 \) can be compared to the difference between \( d_{c2} \) and \( d_{c1} \) because \( \Delta d_1 \) is known exactly from the translation system measurement. If we assume \( d_{c1} \) is the exact object distance (\( d \)), \( d_{c2} \) can be compared with \( d_{c1} + \Delta d_1 \) and \( d_{c3} \) can be compared with \( d_{c1} + \Delta d_1 + \Delta d_2 \). We can obtain the errors and validate the CC method for a single object using this procedure.

A. Two-Dimensional Dots

The calibration target on which the 2D dots are inscribed was used as the 2D test objects. This target is initially fixed at P1 and the exact distance from the target to the CCD is assumed to be \( d \). After the first hologram was recorded, the CCD was moved consecutively to P2, P3, P4, and P5, taking holograms at each position. Since the minimum incremental distance of the translation system is 33 nm, the moving distances of the CCD from P1 are chosen as the multiple of this distance and they are 6.6 (namely, 200.000 \( \times \) 33 nm), 13.2, 46.2, and 79.2 nm, as is shown in Table 1. The five holograms that were taken at each position are shown in Fig. 5.

The positions of the focus planes of the 2D dots were calculated using the above holograms and the CC method. The results are summarized in Table 1. The focus plane of P1 that was calculated by the CC method is assumed to be the exact position of the object (\( d \)). The distances moved by the translation system from the object were compared to the distances calculated by the CC method at the corresponding positions. The differences between those values can be considered the errors of the focus plane determination by the CC method. The results show that the errors of the focus plane determination of the 2D object by the CC method can be acceptable when the object distance is in the interval between 150 and

![Fig. 4. Validation experiments for the CC method for both 2D and 3D objects.](image-url)

![Fig. 3. Correlation coefficient method.](image-url)

### Table 1. Comparison of the Object Distances Moved by the Translation with those Calculated by the CC Method for the 2D Dots

<table>
<thead>
<tr>
<th>Positions</th>
<th>P1 (moved by translation system)</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>from P1</td>
<td>0.00</td>
<td>6.6</td>
<td>13.2</td>
<td>46.2</td>
<td>79.2</td>
</tr>
<tr>
<td>from object</td>
<td>161.07</td>
<td>167.67</td>
<td>174.27</td>
<td>207.27</td>
<td>240.27</td>
</tr>
<tr>
<td>from object</td>
<td>161.07</td>
<td>167.675</td>
<td>174.275</td>
<td>207.285</td>
<td>240.3</td>
</tr>
</tbody>
</table>

Error (\( \mu m \))

|                  | 5 | 5 | 15 | 30 |

*aUnit: millimeters.*
220 mm, which also confirms the previous results [11]. The positioning errors are proportional to the object distance, which also agrees with the situation of the simulation holograms discussed in the previous research [11].

B. Three-Dimensional Droplets

To further simulate the spray droplets, which are our ultimate measurement target, we used droplets generated by an injector needle as the real 3D test objects to validate the CC method for the focus plane determination. The experimental setup is shown in Fig. 6. The CCD is also moved by the PI translation system and an injector needle is fixed on the frame. Minute control of the injector produced different-sized droplets greater than needle diameter (100 μm). This size range is large enough for checking the accuracy of the CC method for the 3D droplet measurements. Using the translation system, the holograms at five positions (P1, P2, P3, P4, and P5) were recorded in the same way as in the 2D case. The holograms of the different-sized droplets at different positions are shown in Fig. 7. The CC curves of one droplet \(D_d = 436.6 \mu m\) are shown in Fig. 8. The shape of the curve looks good and the peaks for the focus plane can easily be obtained. The reconstruction images of this droplet at the focus plane that were calculated by the CC method are shown in Fig. 9.

Using the CC method, the focus planes of all the droplets in Fig. 7 were calculated. The focus plane calculated by the CC method at P1 was assumed to be the first exact distance of the object \(d\). The positions obtained by the CC method and by the translation system are compared in Table 2. The errors in the different-sized droplets at different positions are shown in Fig. 10. The results are consistent with those discussed in the numerical simulation of holograms [11]. The errors are generally proportional to the particle size, but all the errors exist within an acceptable range of tolerance, regardless of the droplet sizes. The errors of the droplets are almost similar to those of the 2D dots. Especially, the errors at the position P5 (∼240 mm), which is beyond the optimal interval of 150 to 220 mm, are much smaller.
than those of the 2D dots. The reason for this improvement is that some beams are refracted at the droplet boundary, resulting in reduced particle details, which makes the correlation more sensitive to defocus distances. This means that the intensity change of the 3D particle reconstruction image along the reconstruction axis is faster than that for the 2D case. This feature can be seen in Fig. 11, which compares the reconstruction images of a 2D dot and a 3D droplet at focus and defocus planes. The distance between focus and defocus planes is chosen as 10 mm, which is the correlation coefficient interval we used for calculation of CC values. It is easy to see that the intensity difference of the 2D dot reconstruction images is smaller than that for the 3D droplet case. This means that the depth of focus of a 2D dot is larger than that of a 3D droplet at the same size. In other words, the shape of the CC curve for 2D dots should be flatter than that of 3D droplets and the peak, i.e., the focus plane of 3D droplets, is easy to obtain. The relationships among the shape of the CC curve, the depth of focus, and the error of the focus plane determination were discussed in our

Fig. 8. (Color online) CC curves of a droplet ($D_d = 436.6\,\mu m$) at different positions: (a) P1, (b) P2, (c) P3, (d) P4, (e) P5.

Fig. 9. Hologram and reconstruction images of the droplet in Fig. 8(a) ($D_d = 436.6\,\mu m$, reconstruction distance = 160.8 mm at P1): (a) hologram, (b) reconstruction image, (c) magnified image of (b).
The means and standard deviations of the errors of the focus plane determination by the CC method for the different sized droplets at P2, P3, P4, and P5 are 173.99 μm, 174.00 μm, 174.00 μm, and 174.00 μm, respectively.

Based on the above results, the CC method works very well in the focus plane determination of the 3D particles, even for large sizes and far distances. Therefore, the CC method can be used as an appropriate tool for determining particle locations in 3D particle fields.

C. Effect of Other Particles

In the real particle measurement field, a test volume is filled with many particles and the distances between these particles are usually very short. Therefore, the effect of other particles on the focus plane determination by the CC method must be discussed.

The effect of other particles existing around the focus plane of the particle of interest is a very difficult concept to express by a mathematical method. So, a simple apparatus was designed to conduct experiments to investigate this kind of influence, as is shown in Fig. 12. Glass beads that are similar to the spray droplets were used as the test target. Their size ranged between 150 and 450 μm, which is also large enough to check the CC method. The beads were pasted on short poles and the distance between them could be adjusted easily by plugging the poles into the different holes of the base. The base was fixed in our hologram recording system and a glass bead was plugged into one hole, and then the first hologram was recorded. After that, other glass beads were plugged into the other holes around the first bead, and holograms of several particles were obtained. In Fig. 13, the hologram B was recorded with the second bead located on the left back of the first bead, the hologram C was recorded containing more beads existing around the first bead, and the hologram D was recorded to check the case of highly overlapped particles. For four holograms, the focus plane of the first bead by the CC method was calculated using the same region of interest (ROI), as is shown in Fig. 13.

![Fig. 10. Errors of focus plane determination for different-sized droplets at different positions.](image1)

![Fig. 11. Comparison of images of a 2D dot and a 3D droplet at different reconstruction distances: (a) 2D dot reconstructed at the focus plane (d = 155.8 mm), (b) 2D dot reconstructed at the defocus plane (d = 165.8 mm), (c) 3D droplet reconstructed at the focus plane (d = 160.8 mm), (d) 3D droplet reconstructed at the defocus plane (d = 170.8 mm).](image2)

Table 2. Comparison of the Object Distances Moved by the Translation with those Calculated by the CC Method for Different-Sized Droplets

<table>
<thead>
<tr>
<th>Positions</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size (μm)</td>
<td>CC</td>
<td>dc1</td>
<td>CC</td>
<td>del + 6.6</td>
<td>CC</td>
</tr>
<tr>
<td>Error (μm)</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>185</td>
<td>160.80</td>
<td>160.80</td>
<td>167.410</td>
<td>167.40</td>
<td>174.005</td>
</tr>
<tr>
<td>Error (μm)</td>
<td>10</td>
<td>5</td>
<td>10</td>
<td>25</td>
<td>0</td>
</tr>
<tr>
<td>303.4</td>
<td>160.81</td>
<td>160.81</td>
<td>167.395</td>
<td>167.41</td>
<td>174.020</td>
</tr>
<tr>
<td>Error (μm)</td>
<td>15</td>
<td>10</td>
<td>15</td>
<td>30</td>
<td>10</td>
</tr>
<tr>
<td>436.6</td>
<td>160.80</td>
<td>160.80</td>
<td>167.405</td>
<td>167.40</td>
<td>174.020</td>
</tr>
<tr>
<td>Error (μm)</td>
<td>5</td>
<td>20</td>
<td>20</td>
<td>30</td>
<td>10</td>
</tr>
<tr>
<td>451.4</td>
<td>160.81</td>
<td>160.80</td>
<td>167.420</td>
<td>167.40</td>
<td>174.025</td>
</tr>
<tr>
<td>Error (μm)</td>
<td>20</td>
<td>25</td>
<td>25</td>
<td>20</td>
<td>10</td>
</tr>
</tbody>
</table>

*Unit: millimeters.*
The CC curves are shown in Fig. 14 and the focus planes are shown in Table 3. The mean and standard deviation of the focus plane of the first bead is 173.118 mm ± 5.6 μm. The errors of four holograms are 8, 2, 7, and 3 μm, respectively. The curves in Fig. 14 show the oscillatory behavior, but the peak can be obtained easily. The oscillatory behavior may come from the effect of twin images (i.e., virtual). In our study, we used the Winner filter to reduce the effect of noises, including noise coming from twin images. But as shown in Fig. 13(c), the twin images cannot be removed clearly. Denis et al. [14] discussed the influence of twin image on reconstruction images and revealed the flexuous behavior of the signal-to-noise ratio (SNR). Based on the magnitude of the errors of the focus plane and the shape of curves, the effect of other particles on the focus plane determination using the CC method seems to be very slight, even though the ROI includes the other particles. Therefore, the CC method can be expected to work well in the analysis of real sprays in which many droplets exist in the test space.

4. Conclusions

Using the high-precision distance movement provided by the PI translation system, validation experiments were conducted in order to check the feasibility of the CC method for the focus plane determination of particles in digital particle holography. The 2D dots of the calibration target, the real droplets generated by an injector, and glass beads are used as the test targets. The results for the 2D dot case are similar to those obtained in previous research by means of simulation holograms. For the real 3D droplets, the CC method can also work well for large-sized droplets, even at large object distances. The simple experimental setup with the glass beads was used to check the effect of other particles on the focus plane determination using the CC method. The results show that the effect is very slight, even though the region of interest includes the other particles. All experimental results prove that the CC method works well in the determination of particle positions in digital particle holography, especially for 3D objects.

References

4. F. Dubois, C. Schockaert, N. Callens, and C. Yourassowsky, “Focus plane detection criteria in digital holography.”

Table 3. Focus Planes of the First Bead in Fig. 13 Calculated by the CC Method

<table>
<thead>
<tr>
<th>Hologram</th>
<th>Focus plane</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>173.125</td>
</tr>
<tr>
<td>B</td>
<td>173.115</td>
</tr>
<tr>
<td>C</td>
<td>173.110</td>
</tr>
<tr>
<td>D</td>
<td>173.120</td>
</tr>
</tbody>
</table>

*Unit: millimeters.*